# Comparing Jeovani, Desean, and Cesar's Strategies for Solving a Multidigit Subtraction Word Problem

This story is a part of the series:

***What's Next?* Stories of Teachers Engaging in Collaborative Inquiry Focused on Using Student Thinking to Inform Instructional Decisions**

# *What's Next?*

## Stories of teachers engaging in collaborative inquiry focused on using student thinking to inform instructional decisions

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## Introduction

In this lesson, students are provided with a subtraction word problem with the numbers 124 and 38. They are asked to solve the problem in any way that makes sense to them as long as they can record their thinking so that another student could understand what they did. After each student in the class had a chance to solve the problem, a group of teachers examined the distribution of strategies used by students. The teachers observed that many students used direct modeling strategies involving groups of ten and devised a plan to encourage students to start using more abstract and efficient mental strategies by studying how they are connected with the direct modeling strategies.

## Relevant Florida Mathematics Standards

*MAFS.2.OA.1.1* Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.

*MAFS.2.NBT.2.7* Add and subtract within 1000, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method. Understand that in adding or subtracting three digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones; and sometimes it is necessary to compose or decompose tens or hundreds.

## Background Information

Consider reading chapter seven of *Children's Mathematics: Cognitively Guided Instruction* (Carpenter et al., 2015). This chapter provides background on student solution strategies for multidigit problems. It also expands on the strategies explained in the *Analyzing Student Thinking* section.

Carpenter, T. P., Fennema, E., Franke, M. L., Levi, L., & Empson, S. B. (2015). *Children's Mathematics: Cognitively Guided Instruction (2nd ed.)*. Portsmouth, NH: Heinemann.

## Analyzing Student Thinking

In the context of a professional-development experience, a group of teachers developed a problem with a goal of assessing second-grade students' understanding of subtraction with multidigit numbers in problem-solving scenarios. The teachers decided on the following problem.

> *Jamie had 124 stickers. She gave 38 of her stickers to her friends. How many stickers did she have left?*

To implement this task, the teacher read the problem to the whole group of students. The students were also presented with the problem in writing with plenty of room to show their work. They were asked to solve the problem in whatever way made the most sense to them. The only requirement was to record their thinking in writing or drawings so that others could make sense of how they solved the problem without any further explanation. While the students worked, they had access to manipulatives (including base-ten blocks and snap cubes) and writing materials. While the students worked on the problem individually, the teacher circulated and provided assistance and help through questioning as needed. The teacher's main goal was to find out how the students would solve the problem provided they understood what was being asked. The teacher therefore used this time to help students who were struggling to get started. The teacher simply aided in comprehension of the problem and did not

> *The teachers recognized that this goal would probably not be reached in one lesson. Instead, they expected it to be a long-term goal that would require considerable time and practice.*

suggest strategies or models.

Afterward, the teachers studied the students' work and sorted students into the following categories based on the strategies they used to solve the problem.[1]

***Multidigit computation strategies***

The student who uses a *direct modeling with ones* strategy uses objects to act out the story or situation in chronological order, representing each of the quantities in the problem as a set of ones using manipulatives or pictures and then counts the objects or pictures to determine the answer. For example, for the problem about Jamie's stickers, a student might create a set of 124 individual cubes, remove 38 cubes from the set, and count the remaining cubes by ones to determine how many stickers were left.

The student who uses a *counting by ones* strategy does so without representing all numbers in the problem. Fingers, objects, or tally marks are often used to keep track of the number of counts. In the problem about Jamie's stickers, the student might start at 124 and count backwards by ones 38 counts to determine how many stickers were left.

Teachers did not anticipate seeing many students use these first two strategies, but they thought it was important to discuss and allow for the possibility that some students might use them.

The student who uses a *direct modeling with tens* strategy follows the story or situation and represents the multidigit numbers in the problem using manipulatives or pictures that reflect the base-ten structure of our number system (e.g., with base-ten blocks or base-ten pictures), then counts the objects or pictures to determine the answer. In the word problem about Jamie's stickers, the student represent 124 as one 100, two tens, and four ones (or use another convenient way to represent 124, such as 12 tens and four ones). The student would then take away three tens and eight ones and count the remaining objects representing stickers by tens and ones after the 38 stickers had been removed. (See Figure 1 for an example of written work cre-

1 The descriptions of strategies presented in this section are the current descriptions used by our team, and we consider them to be fluid, as our understandings of these ideas continue to evolve. For a more detailed discussion of these terms, consider reading Carpenter et al. (2015).

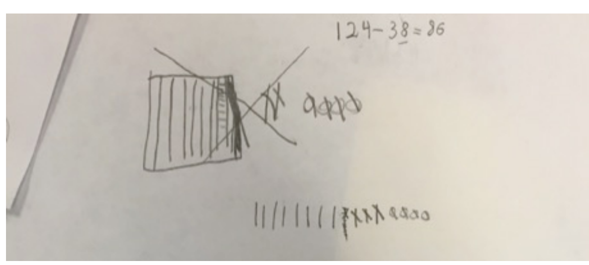


*Figure 1. A student's drawing of a* **direct modeling with tens** *strategy for the Jamie's-stickers problem.*

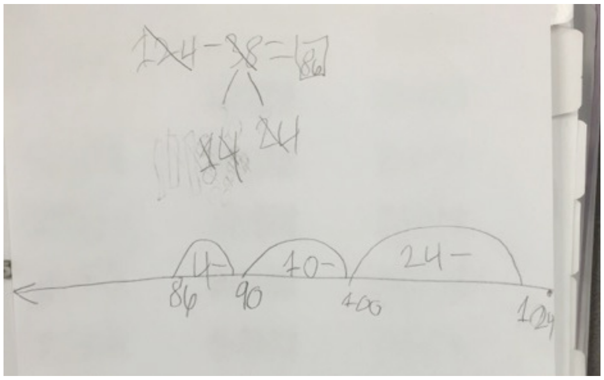


*Figure 2: An example of a student's written representation of an* ***incrementing*** *strategy.*

ated by a student who used a *direct modeling with tens* strategy.)

*Ad hoc ("invented") algorithms*

A student who uses an *incrementing* strategy determines the answer by increasing or decreasing by partial sums or differences. To solve the problem about Jamie's stickers with an *incrementing* strategy, a student might decompose the 38 into parts that are more easily subtracted. For example, Figure 2 illustrates how a student used *incrementing* first to subtract 24 from 124 to get to 100 and then to subtract the remaining 14—decomposed into ten and 4—to determine 86 to be the difference.

A student who uses a *combining the same units* strategy operates on the tens and ones separately, and the partial sums are then combined to yield a final result. To solve the problem about Jamie's stickers with a *combining the same units* strategy, the student might subtract 30 from 120 to get 90, subtract 8 from 4 and think of the result as negative four (or four left still to subtract), and finally subtract 4 from 90 to get 86.

A student who uses a *compensation* strategy adds or subtracts the numbers by adjusting one number to compensate for changes made in another number. To solve the problem about Jamie's stickers, a student using a *compensation* strategy might increase both numbers by two to transform the problem to 126 – 40 and then find the difference. The change in numbers does not change the difference, but it might transform them to an easier problem for the student to solve.

The student might also attempt a strategy that does not clearly fit in to one of the strategies listed above, and the teachers in this group decided to classify these as *other*.

After the teachers sorted the students into strategy categories, they created a chart like the one in Figure 3.

Noticing the large number of students who fell into the *direct modeling with tens* category, the teachers discussed how to help these students learn more sophisticated strategies. They also noticed that many of the students in this group seemed ready to connect their strategies to an *ad hoc* algorithm. For example, they represented 124 with 1 100s block, 2 tens rods, and four ones. They first removed the 24 and then decomposed the 100 to subtract the remaining cubes. The teachers thought that these students would be ready to see this process as a series of equations, such as 124 – 24 = 100 and 100 – 14 = 86. They

| *Direct modeling with ones* | *Counting by ones* | *Direct modeling with tens* | *Incrementing* | *Combining the same units* | *Compensation* | *Other* |
|---|---|---|---|---|---|---|
| Alaysia | Izara | Marquise<br>Hailey<br>Vianca<br>Hayden<br>Paula<br>Jeovani | Desean<br>Cesar | Wyatt (errors)<br>Carmelo<br>Sheryl | | Ivy |

*Figure 3. The sorting of the students by strategy used.*

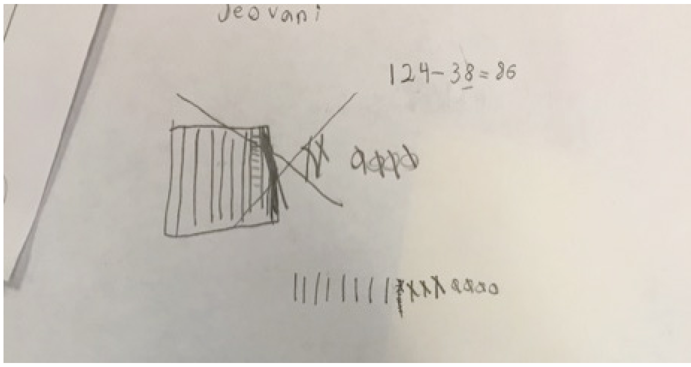

*Figure 4: Jeovani's strategy.*

thought this change might help the students to start using more advanced, abstract strategies such as *incrementing*.

On the basis of this information, the teachers developed the following learning goal for students:

> *On separate result unknown word problems, students will move from direct modeling with tens to ad hoc ("invented") algorithms and will use drawings or other written notation to communicate their thinking.*

The teachers recognized that this goal would probably not be reached in one lesson. Instead, they expected it to be a long-term goal that would require considerable time and practice, but they moved forward with planning an instructional sequence for a single lesson with a focus on this long-term goal.

## Planning For The Lesson

*Rationale for the problem selected*

The teachers decided to plan a lesson around discussion of students' strategies on the Jamie's-stickers problem. Because the goal was to help students using *direct modeling with tens* strategies to connect those strategies with written notation describing related *incrementing* strategies, the teachers decided to spend the time working from the strategies that the students had already used. Building the lesson around one of the problems the students had solved that morning allowed them to (a) build on student thinking they observed the students using on that day, (b) save the instructional time it would take to pose a new problem and use that time to provide opportunity for students to analyze one another's solutions in depth, and (c) permit time for the teachers to think deeply—before the students were present—about how the students' strategies could be notated and to be highly prepared to lead the corresponding discussion with students in the classroom.

The teachers decided the second-grade students would benefit from analyzing and discussing Jeovani's *direct modeling with tens* strategy first (see Figure 4). This strategy was selected to be presented first because the teachers conjectured that it would set the stage with a strategy that most students in this classroom would understand. Moreover, Jeovani was particularly good at explaining his thinking verbally and in writing, and his explanation might help the other students to learn.

They would then have Desean share his *incrementing by tens* strategy (see Figure 5) and, through class discussion, encourage the students to draw connections between this work and Jeovani's strategy. Finally, the teachers decided to showcase Cesar's strategy (see Figure 6), which was selected as a means to provide opportunities for students to draw connections among Jeovani's, Desean, and Cesar's solution strategies and notation.

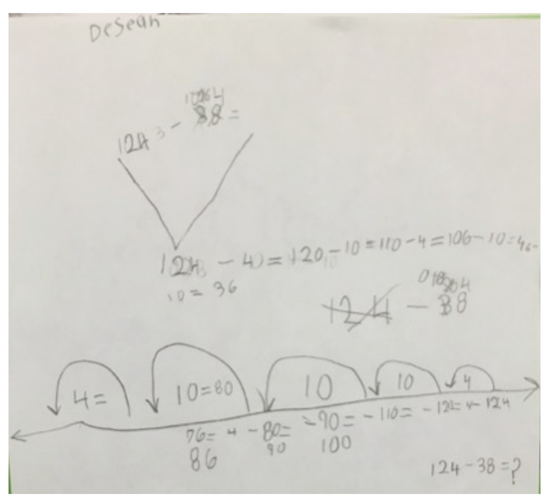

*Figure 5: Desean's strategy.*

The teachers decided that, even though only three students would share their work, as many students as possible should be included during the discussion. The teachers planned to implement a number of turn and talk opportunities to encourage all students to participate in connecting the *direct modeling with tens* work to equations and *incrementing* strategies.

## Lesson Plan

Previously, in the ***Analyzing Student Thinking*** section, the teachers developed the following goal:

> *On separate result unknown word problems, students will move from direct modeling with tens to ad hoc ("invented") algorithms and will use drawings or other written notation to communicate their thinking.*

This lesson was developed to allow three students (Jeovani, Desean, and Cesar) to share their strategies on the Jamie's-stickers problem, which they had previously solved.

1. Begin by reviewing the problem the students completed earlier in the day.

2. Invite Jeovani to come to the front of the class to explain how he solved the problem and record his thinking on in writing for the class to see.

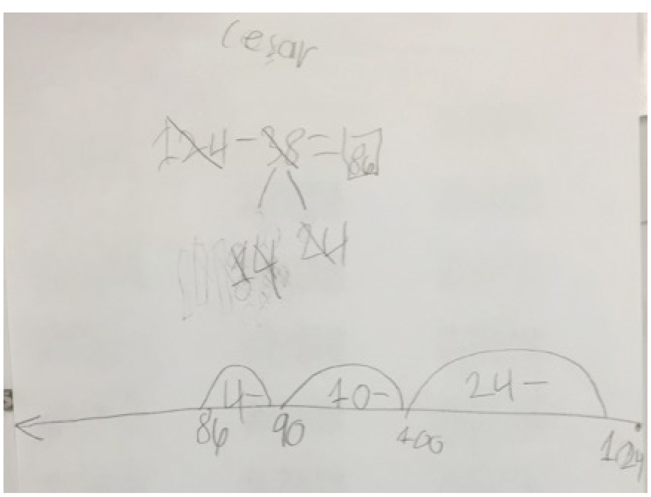


*Figure 6: Cesar's strategy.*

*They would then have Desean share his* ***incrementing by tens*** *strategy and, through class discussion, encourage the students to draw connections between this work and Jeovani's strategy.*

3. As Jeovani shares his work, ask any prompting questions necessary to make his work clear to the class. For example, draw attention to how he changed the 100s block into to ten tens—which is equivalent—to make subtraction easier.

4. After Jeovani shares, ask the students to turn and talk with their neighbors about his solution strategy. Provide guiding questions as necessary to help point out any features in that students might benefit from seeing as related to other strategies. For example, point out how Jeovani first took away the 24. This point will be important to connect as students share *incrementing* strategies in which they first take away the 24 to get to 100.

5. Call the group back together and have students share what they noticed about Jeovani's strategy. Ask plenty of prompting questions to ensure that the students understand Jeovani's strategy and his answer.

6. Next, call Desean to come to the front of the class. Ask Desean to explain how he used the number line to solve the problem and to record his thinking visually for the class (for a

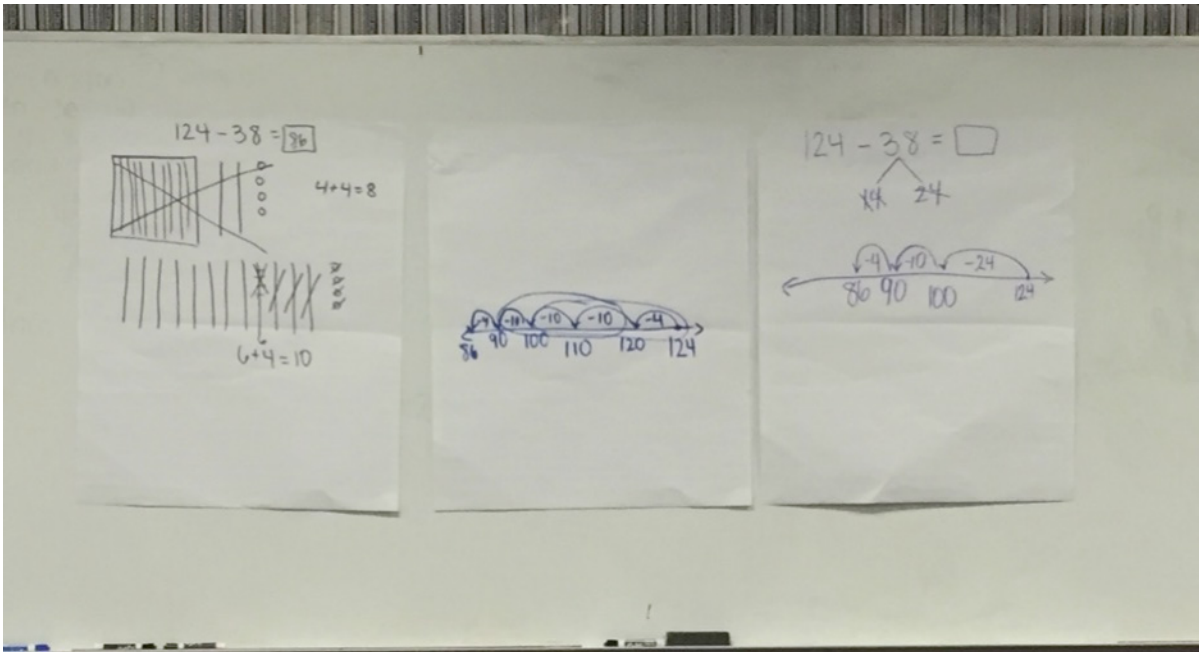


*Figure 7: Jeovani's, Desean's, and Cesar's strategies recreated on chart paper and presented side by side to facilitate comparing and contrasting the strategies.*

visual representation of how the teacher in this group recorded his thinking, see Figure 7.) Consider using the same process that was used with Jeovani.

7. As Desean shares his work, ask any prompting questions necessary to make his work clear to the class. For example, you might ask clarifying questions about how he used the number line to subtract. You might also consider asking questions that allow other students to make connections back to Jeovani's work.

8. Ask the students to turn and talk with their neighbors about Desean's number-line model. Provide guiding questions as necessary to help point out any features in the work to be related to other strategies. For example, point out that he took away groups of ten by moving back on the number line. Doing so may help students see similarities to Jeovani's taking away two groups of ten with the base-ten rods he drew.

9. If the connection has not yet been made, create an opportunity to connect Desean's notation to Jeovani's strategy. In an effort to attain the goal set in the *Analyzing Student Thinking* section, connect a *direct modeling with tens* strategy to an *incrementing* strategy; helping students to make this connection is very important. Try to help the students see that jumping back by ten on a number line is very similar to removing a tens rod when direct modeling by tens.

10. Invite Cesar to the front of the class, and ask him to explain how he used the number line to solve the problem and record his thinking visually for the class.

11. As Cesar shares his work, ask any prompting questions necessary to make his work clear to the class. These may include certain features of the drawing that may be related to previous strategies. Consider asking prompting questions about how Cesar and Desean's number lines are similar and different.

12. Ask the students to turn and talk with their neighbors about Cesar's solution strategy. Ask guiding questions as necessary to help students to see connections among Cesar's strategy and the other two. For example, you may ask the students to pay close attention to the 24 he subtracted first and to consider where they may see that in Jeovani's and/or Desean's work.

13. Call the group back together and have students share what they noticed about Cesar's strategy. Ask plenty of prompting questions to ensure that the students understand this strategy and his answer.

14. Finally, initiate a discussion about the connections among all three strategies. During this discussion, be sure to help the students connect the three strategies.

## Reflection

*What we learned about the students*

In this lesson, nice connections were made between the *direct modeling with tens* strategy and the *incrementing* strategy. When implemented in the classroom, one decision that seemed to make this process work particularly well was to recreate each student's strategy visually for the class. By the end of the lesson, depictions of all three strategies were posted next to each other (see Figure 7). This comparison seemed to help students to understand some of the connections among the strategies.

The teacher in the lesson also asked some very pointed questions that elicited very important student thinking. Two examples included:

*How did you know when you were done?* This question helped students to rephrase what they had already done and how they knew when they were finished. It also helped the individual student sharing to be sure he was explaining his work clearly. The teacher used this technique in an effort to provide the students with time and opportunities to make sense of the strategy.

*How much more do you need to subtract?* This question and minor variations of it throughout the lesson helped the students make connections among the strategies. For example, the teacher asked this question after Desean had jumped back 24 (four ones and two tens) to 100. After Desean answered 14, the teacher then asked the students to find where the 14 was in Jeovani's strategy.

Some key suggestions arose about how to make the lesson even better in the future. The teachers suggested using two bright colors to highlight the contrast between the 24 and the 14 in Cesar's strategy. Doing so would allow the teacher to use those same colors when connecting Cesar's work to Jeovani's work.

*On the basis of the outcomes from this lesson, what are the ideas for the next lesson?*

The teachers agreed even before the lesson that the students would probably need more opportunities to solve separate-result-unknown problems with multidigit numbers. After reflecting, they agreed that some students seemed to make the connection between *direct modeling with tens* strategies and *incrementing*, but additional opportunities were going to be necessary for all students to make this transition.

The teachers thought that providing more separate-result-unknown problems would allow the students more opportunities to use *incrementing* strategies when solving subtraction problems. Separate-result-unknown problems are the easiest type of subtraction problems, so the problem type does not get in the way of thinking about how to model the problem or whether the numbers should be added or subtracted.

In addition, the teachers discussed using different numbers in the future. They discussed the types of, and pairs of, numbers that might help students to begin using *incrementing* strategies. In general, they decided that asking students to solve some problems where the amount being subtracted is a multiple of ten would be an interesting next step. For example, the problem 112 – 30 provides multiple opportunities to use *incrementing*. Students might first subtract 12 to get to 100 and then subtract the remaining 18. Or they might subtract ten from 112 to get 102, then ten more to get to 92, and the subtract the remaining ten to get 82.

# *What's Next?*

## Stories of teachers engaging in collaborative inquiry focused on using student thinking to inform instructional decisions

*What's Next?* is a collection of stories documenting professional development experiences shared by elementary teachers working collaboratively to study the complex process of teaching and learning mathematics. Each story in the collection describes practicing teachers studying the thinking processes of real students and using what they learn about those students to make decisions and try to help advance those students' understanding on that day.

The teachers in the stories start by learning about how individual students solve a set of mathematics problems. They use this freshly gathered knowledge of student thinking to develop near-term learning goals for students and a lesson plan tailored to specific students on that specific day. One of the teachers implements the planned lesson while the others observe in real time. The teachers then gather to discuss and reflect on their observations and insights.

In these lessons, the practice of teaching is slowed way down. The stories tell of teachers who are studying student thinking and using that information to plan and implement instructional decisions at a pace that is much slower than is typical in daily practice. They depict a process with many aspects in common with formative assessment and lesson study, both of which are also conceptualized as processes and not outcomes.

The stories depict real situations that occurred in real time and include both successes and shortcomings. We hope they will be studied and discussed by interested educators so that the lessons and ideas experienced by these teachers and instructional coaches will contribute to additional learning and sharing among other interested teachers.

Learn more about these and other stories at **http://www.teachingisproblemsolving.org/**